HYDRAULIC CHARACTERISTICS AND VAPOR CONTENT IN A CHANNEL WITH BOILING OF A LIQUID UNDERHEATED UP TO THE SATURATION TEMPERATURE

B. R. Bergel'son and A. S. Gerasimov

UDC 536.423.1

A method is proposed for calculating the hydraulic characteristic and vapor content in a channel with underheated boiling.

In the present report we discuss a semiempirical model for describing the hydraulic characteristic of a channel in which surface boiling of a liquid occurs at relatively low pressures: $\Delta P = \Delta P(X)$. By X we understand the set of thermophysical and geometrical characteristics of the heat-transfer agent and the channel. The model, in which only one empirical parameter is used, also proved to be very useful for determining the true volumetric vapor content in the channel. On the basis of this model equations are obtained in the report which are convenient for making engineering calculations of channels of heat exchangers where surface boiling plays an important role.

The channel, at whose entrance water enters with $i_{en} < i_s$, can be divided arbitrarily into four sections differing in the character of the processes of heat transfer and of accumulation of heat in the heat-transfer agent. We characterize all four sections of the channel and define their boundaries following the standard terminology.

Heating Section

An increase in the enthalpy of the liquid from i_{en} to i_1 occurs in this section of the channel with a length z_1. At an enthalpy i_1 the temperature of the heating surface exceeds the saturation temperature by an amount $\Delta t_v = t_w - t_s$; Δt_v is the minimum superheating of the heat-transfer surface at which the generation of vapor bubbles is possible. In accordance with such a definition,

$$i_1(z_1) = i_s(z_1) + \Delta t_v C_p - \frac{N\Psi(z_1) C_p}{H\alpha(z_1)\Pi_t}. \quad (1)$$

Here $\Psi(z)$ is the distribution of energy release over the length of the channel. To estimate the quantity Δt_v we use the concepts developed in [1]. Within the limits of the viscous sublayer Δ the variation of the liquid temperature obeys a linear law and can therefore be represented as

$$t'(Y) = t_l + \frac{q}{\alpha} - \frac{q}{\lambda} y, \quad (2)$$

where y is the distance from the heating surface ($0 \leqslant y \leqslant \Delta$); t_l is the balanced temperature of the liquid at the point z_1. We designate the critical radius of a vapor bubble generated at the heating surface as y. This quantity and the superheating $\Delta t'_v$ of the liquid near the heating surface are connected by the relation

$$\Delta t'_v = 2\frac{\sigma T_s}{r\rho'' y} = \frac{A}{y}. \quad (3)$$

The development of a vapor bubble at the heating surface is possible only in the case when the liquid temperature t'(y) near this surface, determined by Eq. (2), satisfies the condition (3). Since $\Delta t'_v = t'(y) - t_s$, substituting (2) into (3) and designating $\Delta t = t_s - t_l$, we obtain

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 37, No. 5, pp. 784-792, November, 1979. Original article submitted February 23, 1979.

0022-0841/79/3705-1267$07.50

$$\frac{q}{\lambda} y^2 + \left(\Delta t - \frac{q}{\alpha}\right) y + A = 0. \quad (4)$$

The roots of Eq. (4) determine the minimum and maximum radii of vapor bubbles which can form at the heating surface at given values of q, t_s, t_l, and α. If we set

$$\frac{1}{4}\left(\frac{\lambda}{\alpha} - \frac{\Delta t \lambda}{q}\right) = \frac{\lambda}{q} A, \quad (5)$$

then the two roots of Eq. (4) degenerate into one root y_0. In this case the critical radius of a vapor bubble is

$$y_0 = \left(\frac{\lambda}{q} A\right)^{1/2}. \quad (6)$$

Substituting (6) into (3), we obtain the minimum superheating of the liquid near the heating surface at which the formation of vapor bubbles begins. As for the minimum superheating of the heat-transfer surface, of interest to us, in accordance with (2), (3), and (6) we obtain

$$\Delta t_v = t_w - t_s = 2\left(\frac{qA}{\lambda}\right)^{1/2} = 2\left(\frac{q\sigma T_s}{r\gamma''\lambda}\right)^{1/2}. \quad (7)$$

Equation (7) gives fully reasonable values of Δt_v. In particular, with q = 1.16 MW/m^2 and T = 373°K we have Δt_v = 15.2°C for water, which corresponds to a vapor bubble radius $y_0 = 4.3 \cdot 10^{-3}$ mm.

Section of Development of Surface Boiling

In this section with a length $z_2 - z_1$ the temperature of the heating surface exceeds the saturation temperature by an amount $\Delta t_v(z) > \Delta t_v(z_1)$, making possible the formation and growth of vapor bubbles. However, the underheating of liquid in the core of the stream is still so great that all the vapor formed condenses without being able to go beyond the limits of the thin boundary layer. In the section of surface boiling the average volumetric vapor content over a channel cross section is practically equal to zero and, as follows from this, the liquid velocity does not depend on z. The upper limit z_2 of the section corresponds to the balanced stream enthalpy i_2 at which equilibrium between the intensities of vapor formation and condensation is disrupted. Let us attempt to determine the dependence of i_2 on the power, the channel geometry, and the parameters of the stream of heat-transfer agent. We will assume that the amount of vapor formed at the heating surface at the point z ($z > z_1$) is

$$Q_v(z) \sim \frac{i(z) - i_1}{i_s - i_1},$$

while the amount of vapor which can be condensed at the point z is

$$Q_c(z) \sim \frac{i_s - i(z)}{i_s - i_1}.$$

It follows from the definition that $i = i_2$ when $Q_c = Q_v$. From this condition, using (1), we obtain

$$i_2(z_2) = i_s - \frac{C_p}{2}\left[\frac{q}{\alpha} - \Delta t_v(z_1)\right]. \quad (8)$$

The expression for i_2 can also be obtained in a different way by considering the balance between the vapor formation and condensation at the level of an individual vapor bubble. Approximating the shape of a vapor bubble by a hemisphere and writing the condition of equality of the amount of heat going into the evaporation of water and released in the condensation of vapor [2], we can obtain an expression analogous to (8). In reports on the investigation of vapor content during surface boiling empirical dependences are usually used to determine i_2; in [3], e.g.,

$$i_2 = i_s - 0.35\frac{qC_p}{\alpha}, \tag{9}$$

and in [4],

$$i_2 = i_s - 10^{-6}\frac{qC_p}{W}(14 + 10^{-6}P). \tag{10}$$

Calculations show that (8) and (9) give close values of $\Delta i_2 = i_s - i_2$. Henceforth we will use (8) to determine i_2, representing this dependence in the form

$$i_2 = i_s - \varkappa\frac{N\Psi(z)}{G}.$$

The explicit form of $\varkappa$ as a function of the stream parameters and the channel geometry can easily be obtained from (8).

Section of Developed Surface Boiling

In the section of developed surface boiling vapor formation predominates over condensation. The intensity of the latter declines with greater distance from the start of the section. The essence of the proposed model, lying at at the basis of the further discussion, consists in the introduction of some function $0 \leqslant u(z) \leqslant 1$ which determines at the point z the fraction of the linear power of the channel (the power per unit length) expended on the formation of the noncondensing part of the vapor. At the upper limit of the section of developed surface boiling thermodynamic equilibrium of the two-phase stream is reached: The true enthalpy of the liquid becomes equal to i_s. The balanced enthalpy at the point z_3 is

$$i_3 = i_{en} + \frac{N}{G}\int_0^{z_3}\Psi(z)\,dz, \quad z_3 = z_2 + \frac{m+1}{m}\frac{G\Delta i_2}{N}. \tag{11}$$

On the basis of obvious considerations we can formulate the requirements which the function u(z) must satisfy: 1) $u(z) = 0$ at $z \leqslant z_2$; 2) $u(z) = 1$ at $z \geqslant z_3$; 3) u(z) is a monotonically growing function in the interval $z_2 \leqslant z \leqslant z_3$. A function $u(z_0)$ satisfying these requirements can be represented in the form

$$u(z) = \begin{cases} 0 & \text{at} \quad z \leqslant z_2 \\ u_0\left[\dfrac{i(z) - i_2}{i_s(z) - i_2}\right]^m & \text{at} \quad z_2 \leqslant z \leqslant z_3 \\ 1 & \text{at} \quad z \geqslant z_3. \end{cases} \tag{12}$$

When the energy release is constant over the length of the channel,

$$u(z) = u_0\left(\frac{N}{G\Delta i_2}\right)^m (z - z_2)^m \quad \text{at} \quad z_2 \leqslant z \leqslant z_3. \tag{13}$$

To determine u_0 we use the fact that the true enthalpy of the liquid at the point z_3 is i_s. Since $u(z) \equiv 1$ at $z \geqslant z_3$, we can obtain the following expression for u_0 [5]:

$$u_0 = \left(\frac{m}{m+1}\right)^m\left\{1 + \frac{1}{m} - \frac{m+1}{m(z_3 - z_2)^{m+1}}\int_{z_2}^{z_3}\left[\frac{i_s(z_3) - i_2}{i_s(z) - i_2}\right]^m (z - z_2)^m\,dz\right\}^m.$$

If the variation of i_s over the section $z_2 - z_3$ is small, then

$$u_0 = \left(\frac{m}{m+1}\right)^m.$$

Before turning to the determination of m, we write the equation of motion of a two-phase stream in a vertical channel in the one-dimensional approximation:

$$-s\frac{dp}{dz} = \tau\Pi_h + \frac{d}{dz}[G'W' + G''W''] + \bar{\gamma}g.$$

By integrating this equation we obtain the expression for the total pressure drop in the channel:

$$\Delta P = \Delta P_\zeta + \Delta P_a + \Delta P_g. \tag{14}$$

The three terms on the right side of (14) determine the pressure drops due to friction of the stream against the channel wall, the acceleration of the stream, and the force of gravity. If we express τ through the coefficient of friction ζ, then

$$\Delta P_\zeta = \Delta P_{\zeta 0}\left\{ z_1 + \int_{z_1}^{z_2} \frac{\zeta_1(z)}{\zeta_0}\, dz + \int_{z_2}^{1} \frac{\zeta_2(z)}{\zeta_0}\left[\frac{\gamma'}{\bar{\gamma}(z)}\right]^n dz\right\}; \tag{15}$$

n = 1 corresponds to a homogeneous model of a two-phase stream and n = 2 to separation flow in which the liquid phase moves with the average stream velocity near the channel wall [7].

In the section of development of surface boiling the processes of vapor formation and condensation taking place in a thin boundary layer lead to additional hydraulic losses, which are allowed for in (15) through the effective coefficient of frction $\zeta_1(z)$. The dependence of this coefficient on the stream parameters and the channel geometry was obtained in [6]. In some cases the use of ζ_1 instead of ζ_0 can give an appreciable correction to ΔP_ζ.

The accuracy of the determination of the component ΔP_ζ in the section of developed surface boiling depends on how much the adopted model corresponds to the structure of the two-phase stream in the interval of the parameters under consideration. It is known that the homogeneous representation gives the best approximation in the bubble mode of flow, for which small vapor contents are characteristic. Various modifications of the model with a separate description of the phases [11] prove to be preferable in the projectile and annular modes. In the case of high mass velocities the homogeneous approximation has advantages over other models even at relatively large values of X, particularly in comparison with the Martinelli–Nelson semiempirical method. Experimental data for a steam–water mixture at $W_\gamma' > 2$ kg/m^2. sec (see Figs. 2 and 3 from [11]) prove to be closer to the results of a calculation by the homogeneous model than to those of calculations by the Martinelli–Nelson method in a wide interval of variation of X. For the problem under consideration the choice of the homogeneous approximation to describe the pressure losses to friction at $z \geq z_2$ is based on the following: 1) The use of the homogeneous approximation is the most fully justified in that region of the parameters which is of interest to us (negative X, relatively high W_γ'); 2) while it gives understated values of ΔP_ζ, the homogeneous approximation gives a basically correct description of the dependence of this quantity on the stream parameters, including the vapor content; 3) the simplicity and clarity of the homogeneous approximation allow us to obtain simple equations which can also be extended to the case of other models without particular difficulty. Within the framework of the homogeneous approximation the coefficient of friction $\zeta_2(z)$ is determined by the well-known relations for a one-phase stream, including the Reynolds number

$$\mathrm{Re} = \frac{G d_h}{\mu(X) s}.$$

Under conditions of boiling with underheating and low pressures, $X \ll 1$ and hence $\mu(X) \approx \mu$ [11].

In a channel with a constant cross section and with $\gamma \ll 1$ the pressure losses to stream acceleration are

$$\Delta P_a = \Delta P_{a_0}\left[\frac{\gamma'}{\bar{\gamma}(1)} - 1\right]. \tag{16}$$

The pressure losses due to the force of gravity are

$$\Delta P_g = \gamma' g\left\{ z_2 + \int_{z_2}^{1} [1 - \varphi(z)]\, dz \right\}. \tag{17}$$

By substituting (15)-(17) into (14) we find the dependence $\Delta P(G)$ of interest to us. If we set $\Psi(z) = \text{const}$, $\zeta_2 \approx \zeta_1 \approx \zeta_0$, and γ'', $i_s \approx \text{const}$ in the section from z_2 to 1, then without allowance for the component due to the force of gravity, we obtain

$$\Delta P(G) = \zeta_0 \frac{H}{d_h} \frac{G^2}{2s^2\gamma'} \left[1 + \left(\frac{m}{m+1}\right)^{m+1} \frac{1}{m(m+2)\gamma\varkappa^m} \frac{N}{Gr} \times \right.$$
$$\left. \times \left(1 + \varkappa - \frac{G\Delta i}{N}\right)^{m+2}\right] + \frac{G^2}{s^2\gamma'} \left(\frac{m}{m+1}\right)^{m+1} \frac{1}{m\gamma\varkappa^m} \frac{N}{Gr} \left(1 + \varkappa - \frac{G\Delta i}{N}\right)^{m+1}, \quad (18)$$
$$\varphi(z) = \left\{1 + \left[\frac{Gr}{N}\left(\frac{m+1}{m}\right)^{m+1} \frac{m\varkappa^m}{\left(z + \varkappa - \frac{G\Delta i}{N}\right)^{m+1}} - 1\right]\gamma\right\}^{-1}.$$

Among the few experimental investigations devoted to studying the hydraulics of channels under conditions of surface boiling at nearly atmospheric pressures we can note [8]. The authors made a large number of measurements of pressure losses during water circulation under conditions of surface boiling in slot and round channels with $H/d_t \sim 100$ and a heat load of 100-300 W/cm^2. On the basis of the large amount of experimental material the authors draw an important conclusion: For a channel with a given H/d_t the dependence $\Delta P = \Delta P(G)$ has a minimum at a certain value of the complex

$$\nu = \frac{N}{G_{\min}(i_s - i_{en})}, \quad (19)$$

G_{min} being the weight flow rate of water through the channel corresponding to the minimum of the hydraulic characteristic. The experimental data are well described by the empirical equation

$$\nu = 0.71 + 0.961 \cdot 10^{-3} \frac{H}{d_t} - 0.936 \cdot 10^{-6} \left(\frac{H}{d_t}\right)^2. \quad (20)$$

We can use (20) to determine the parameter m. For this purpose we differentiate Eq. (18) with respect to G, neglecting the relatively weak dependence of the parameter $\varkappa$ and the coefficient of friction ζ_0 on the flow rate. If we now equate $d(\Delta P)/dG$ to zero then we obtain the condition for a minimum of the hydraulic characteristic at which the complex $N/G_{min}\Delta i$ is constant. The substitution of (20) into $d(\Delta P)/dG = 0$ gives

$$1 + \frac{\left(\frac{m}{m+1}\right)^m \Delta i (\nu + \nu\varkappa - 1)^{m+1}}{(m+1)(m+2) r 2\gamma (\nu\varkappa)^m} \left\{1 + \varkappa - \frac{m+3}{\nu} + \frac{2d_h}{\zeta_0 H}(m+2)\left(1 - \frac{m+1}{\nu + \nu\varkappa - 1}\right)\right\} = 0. \quad (21)$$

Equation (21) can be used to find m.

For short channels ($H/d_t \sim 100$) and low pressures, m is considerably larger than one, so that Eq. (21) is considerably simplified:

$$1 - \frac{1}{\zeta_0\gamma}\left(\frac{m}{m+1}\right)^m \frac{\Delta i}{r} \frac{d_h}{H}\left(1 + \frac{1}{\varkappa} - \frac{1}{\varkappa\nu}\right)^m = 0. \quad (22)$$

For large H/d_t and relatively high pressures (P > 20), at which the pressure losses in overcoming the hydraulic resistance of the channel are as large as or larger than the pressure losses in stream acceleration, Eq. (22) will be incorrect and (21) must be used to determine m.

The equations presented above are used to calculate the hydraulic characteristics and vapor content in channels with surface boiling. Calculated hydraulic characteristics and experimental points from [8] are presented in Figs. 1 and 2. The experimental installation on which the test data were obtained consisted of a circulation loop with a working section in the form of a flat slot (3.23 × 25.4 mm). Heat loads in the range of 82-250 W/cm^2 were produced by heating the working section with a low-voltage current. The water flow rate was measured by two independent means: using a flow-meter washer and with a turbine flow meter. The pressure drop in the working section and the pressure at its entrance were measured with mercury manometers. The oscillations of the flow rate were also recorded. The signal was brought out to an oscillograph screen and photographed.

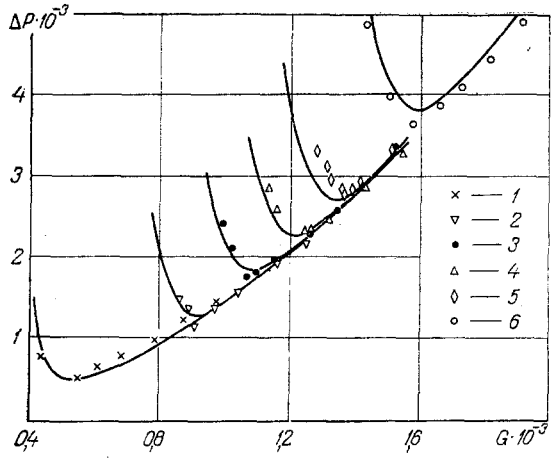

Fig. 1. Dependence $\Delta P = \Delta P(G)$ (curves: calculation; 1-6: test data of [8]; constant heat load along channel length): 1) $q = 82$ W/cm^2; 2) 136; 3) 160; 4) 184; 5) 200; 6) 250. $\Delta P \cdot 10^{-3}$, kg/m^2; $G \cdot 10^{-3}$, kg/h.

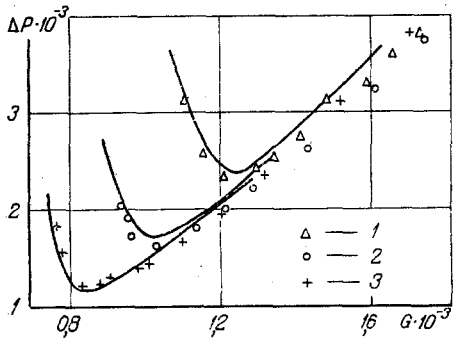

Fig. 2. Dependence $\Delta P = \Delta P(G)$ (curves: calculation; points: experiment; variable heat load along channel length): 1) q_o = 184 W/cm^2; 2) 56; 3) 138.

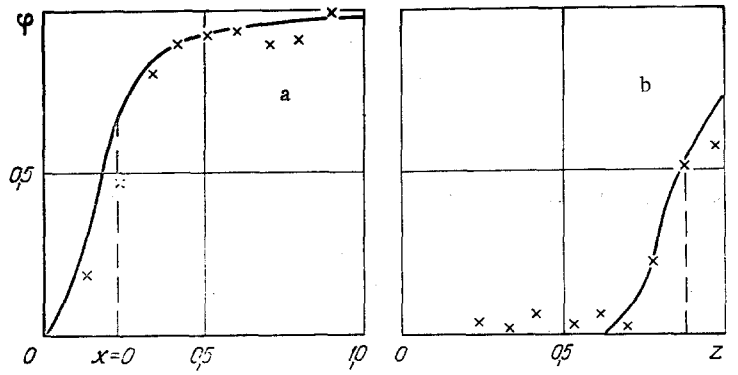

Fig. 3. Dependence $\varphi = \varphi(z)$ (curves: calculation; points: test data of [9]): a) $q = 54$ W/cm^2, $w_j = 53$ g/cm^2·sec, $\Delta i = 46$ kcal/kg; b) $q = 35$ W/cm^2, $w_j = 100$ g/cm^2·sec, $\Delta i = i_s - i_{en} = 60$ kcal/kg.

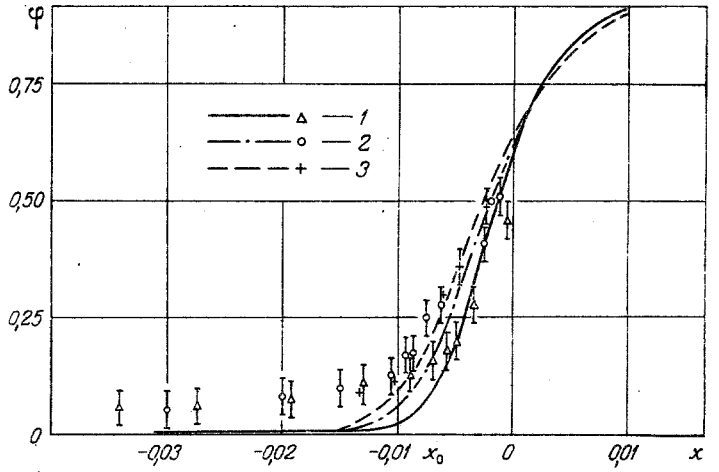

Fig. 4. Dependence $\varphi=\varphi(x)$ (curves: calculation; points: test data of [10]): 1) W = 1.0 m/sec, q = 41.1 W/cm^2; 2) 0.833 and 41.3; 3) 0.666 and 40.0.

In comparing the results of the calculations and experiments presented in Figs. 1 and 2 we can state that at low pressures and in a wide range of heat loads the calculated hydraulic characteristics well describe the corresponding experimental points.

In the method of calculating $\Delta P(G)$ presented above we used one empirical parameter ν, which is determined from Eq. (20). The parameter establishes a one-to-one connection between the power of the channel, the weight flow rate under conditions of a minimum, the hydraulic characteristic, and the channel geometry. From this point of view it is interesting to compare the results of the calculations with experimental data for more than $\Delta P(G)$ alone. For surface boiling an important characteristic, independent of $\Delta P(G)$, is the distribution over the length of the channel of the true vapor content φ in the region of negative values of x. Calculated curves of $\varphi=\varphi(z)$ and $\varphi=\varphi(x)$ and experimental points for pipes [9] and annular slots with internal heating [10] are shown in Figs. 3 and 4. Good agreement is observed between the calculated dependences and the experimental data, obtained through gamma radiography of the test sections over their entire length.

A comparison of the calculated results with the experimental data permits confidence that the method presented above for calculating the hydraulic characteristics and vapor content under conditions of boiling with underheating is suitable for channels of different geometries at low and moderate pressures. Actually, the only limitation on the use of the method is the fulfillment of the condition of nonuniqueness of the hydraulic characteristic of the channel.

NOTATION

H, heated length of channel; d_t, thermal diameter; d_h, hydraulic diameter; Π_t, thermal perimeter; Π_h, hydraulic perimeter; s, through cross section; N, power of channel; q, heat flux; G, weight flow rate; W, velocity; α, coefficient of heat transfer; t, T, temperatures; Δt, temperature difference; i, enthalpy; P, pressure; ΔP, pressure drop; x, weight vapor content; φ, volumetric vapor content; C_p, heat capacity; r, heat of vaporization; λ, coefficient of thermal conductivity; σ, coefficient of surface tension; γ', liquid density; γ'', vapor density; g, acceleration of gravity; $\Delta i = i_s - i_{en}$; ζ_0, coefficient of friction of liquid against the channel wall; τ, shear stress; $\bar{\gamma}$, density of two-phase medium averaged over a channel cross section; μ, dynamic viscosity. Indices: s, on saturation line; en, at channel entrance; l, liquid; v, vapor; w, wall.

LITERATURE CITED

1. H. H. Hsu, "On the size range of active nucleation cavities on a heated surface," J. Heat Transfer, 91, No. 3, 207-216 (1962).
2. N. W. Snyder and T. T. Robin, "A mass-transfer model in subcooled nucleate boiling," J. Heat Transfer, 91, No. 3 (1969).

3. É. A. Zakharova et al., "Calculation of the true vapor content in heated channels," Teploenergetika, No. 6 (1970).
4. R. W. Bowring, "Physical model based on bubble detachment and calculation of steam voidage in the subcooled region of a heated channel," HPR-10 (December, 1962).
5. B. R. Bergel'son and A. S. Gerasimov, Preprint No. 17, Inst. Teor. Eksp. Fiz. (1977).
6. V. A. Knyazev, "Hydraulic resistance in channels with surface boiling," At. Energ., 38, No. 1 (1975).
7. L. Ya. Kramerov and Ya. V. Shevelev, Engineering Problems of Reactor Construction [in Russian], Gosatomizdat, Moscow (1966).
8. R. Forgan and R. Whittle, "Pressure-drop characteristics for the flow of subcooled water at atmospheric pressure in narrow heated channels. I and II," AERE-M1739 (1966).
9. I. K. Ferrell, "A study of calculation on boiling inside a channel," North Carolina State Univ., Raleigh (September, 1946).
10. L. Cimorelli and A. Premoli, CNEN Energ. Nucl., 13, No. 1 (1966).
11. G. B. Wallis, One-Dimensional Two-Phase Flow, McGraw-Hill, New York (1969).

TURBULENT-FLOW RESISTANCE FOR SOLUTIONS OF POLYMERS AND MICELLE-FORMING SURFACTANTS

I. L. Povkh, V. G. Pogrebnyak, and A. I. Toryanik

UDC 532.517.4

Calculation method and results are presented for the hydrodynamic-resistance coefficients of solutions of polymers and surfactants.

There are various methods of reducing resistance such as boundary-layer suction and injection, vibration of the surface with an appropriate frequency and amplitude, etc., but in recent years considerable attention has been given to the use of polymers and micelle-forming surfactants in turbulent flows. Many different points of view have been expressed [1-10] on the mechanism.

On our view, the reduced hydrodynamic resistance in solutions of polymers and surfactants is due to the anisotropy in the viscosity arising in the viscous sublayer and the transitional layer on account of deformation and orientation of the macromolecules and micelles along the flow, i.e., the differences in resistance to the displacement and growth of turbulent eddies in the various directions. The elevated shear viscosity in the perpendicular direction results in additional resistance to pulsation and therefore reduces the general level of mixing. Therefore, the pulsation frequency in polymer and surfactant solutions tends to be lower than that in the solvent at the same Reynolds number, i.e., the generation of turbulence is reduced and the turbulent dissipation in the flow is lower.

Here we present calculations on this basis for the reduction in turbulent friction for polymers and surfactants, and a comparison is made with experiment.

The following formula [11] is used to derive the resistance law for polymers and surfactants:

$$\Delta P_0 = \lambda \frac{l}{2R} \frac{\rho \bar{U}^2}{2} . \qquad (1)$$

Formula (1) relates the pressure difference ΔP_0 across a cylindrical tube of length l and radius R to the resistance coefficient λ, the geometrical dimensions of the part of the tube, the density ρ, and the mean flow velocity $\bar{U}$. We substitute in (1) the Reynolds number $Re = 2R\bar{U}/\nu$ (ν is the kinematic viscosity) to get that the pressure difference in laminar flow is given by

$$\Delta P_{0l} = A \rho Re^2 \lambda_l \nu^2, \qquad (2)$$

Donets University. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 37, No. 5, pp. 793-797, November, 1979. Original article submitted May 17, 1979.

 0022-0841/79/3705-1274$07.50